647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov.- 2024 (Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one: (07)

1. Define: Equivalence class of an element. Show that the relation of congruence modulo m on the set $\mathbb{Z}$ is an equivalence relation.
2. Define: Direct product of two Lattices. Prove that direct product of two lattices is also a Lattice.

Que.1(B) Answer any one: (04)

1. Define: Lattice Isomorphism. In usual notation, prove that: $(S_{30}, D) \cong (P(A), \subseteq)$, where A = {a,b,c}.
2. Let $\mathbb{Z}$ be the set of integers. Define R = {(a,b): b = a^r, for some positive integer r}. Show that ($\mathbb{Z}$, R) is a partially order set.

Que.1(C) Answer any one: (03)

1. Define: Complemented Lattice. Give an example of a bounded lattice which is not a complemented Lattice.
2. Define: (i) Comparable elements in a Poset.
   (ii) Totally ordered set with illustration.

Que.2(A) Answer any one: (07)

1. In usual notation, show that $(P(A), \cap, \cup, ', 0, 1)$ is a Boolean Algebra, where $A = \{a, b, c\}$.
2. State and prove Stone's representation theorem in Boolean Algebra.

Que.2(B) Answer any one: (04)

1. Define: Maxterm. Express $\alpha(x_1, x_2, x_3) = x_2 x_3$ as the product of sum canonical form.
2. Define: Cube array representation of Boolean function. Find cube array representation of $f(x, y, z) = xy + xz'$.

Que.2(C) Answer any one: (03)

1. If $(B, *, \oplus, ', 0, 1)$ is a Boolean Algebra and $a, b, c \in L$ then prove that $(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$
2. If m_i and m_j are distinct minterms in n-variables $x_1, x_2, \dots, x_n$ then prove that $m_i * m_j = 0$.

Que.3(A) Answer any one. (07)

1. Define: Derivative of complex function at z_0. If the complex function $f(z) = u + iv$ is analytic at point $z_0 = x_0 + iy_0$ then prove that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
2. Find analytic function $f(z) = u + iv$ such that $u - v = x + y$.

Que.3(B) Answer any one. (04)

1. If $f(z)$ is an analytic function then in usual notations, prove that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$.
2. Define: Harmonic function and show that $e^x cosy$ is harmonic function and find its conjugate harmonic function.

Que.3(C) Answer any one. (03)

1. (i) Define: Entire function

   (ii) The real part of $f(z) = cosz$ is ….

   (iii) Prove that $f(z) = e^z$ is an entire function.

2. If $f(z) = x^2 + ayx + by^2 + i(x^2 + cxy + dy^2)$ is analytic then Find the values of a, b, c & d.

Que.4(A) Answer any one. (07)

1. Derive C-R conditions for an analytic function in polar form.
2. State and prove Cauchy's fundamental theorem of integration.

Que.4(B) Answer any one. (04)

1. In usual notation, state and prove M-L inequality.
2. Find $\int_c \frac{dz}{\left(z-\frac{2}{z}\right)}$, where $C : |z + 1| = 2$.

Que.4(C) Answer any one. (03)

1. Prove that u = r^2 sin2θ is a harmonic function and also find its conjugate.
2. Find $\int_i^{1+i}(x - y + ix^2)dz$.

Que.5(A) Answer any one. (07)

1. State and prove Liouville's theorem.
2. State Cauchy's inequality and state and prove Morera's theorem.

Que.5(B) Answer any one. (04)

1. Prove that $\int_c \frac{e^{kz}}{z} dz = 2\pi i,\ C :\ |z| = 1$ and hence deduce that:

   (i) $\int_0^{\pi} e^{kcos\theta} \cos(ksin\theta)\, d\theta = \pi$

   (ii) $\int_{-\pi}^{\pi} e^{kcos\theta} \sin(ksin\theta)\, d\theta = 0$

2. If $g(z_0) = \int_c \frac{4z^2+z+5}{z-z_0} dz,\ C :\ |z| = 2$ then find $g(1), g(-1), g(i),\ g'(1), g'(-i)$.

Que.5(C) Answer any one. (03)

1. (i) Define: Smooth arc

   (ii) State Cauchy inequality

   (iii) If $C : z - z_0 = r_0 e^{i\theta}$ then $\int_c \frac{dz}{z-z_0} = \cdots$

2. Evaluate: $\int_c \frac{e^2 dz}{(z-4)(z-2)}$, $C : |z| = 3$

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